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REVIEW OF THE FRENCH EDITION OF HALSTED'S "RATIONAL GEOMETRY."

By JOHN A. EIESLAND, University of West Virginia.

Rational Geometry. By George Bruce Halsted. Second Edition. New York and London: John Wiley and Sons. 1907. Pp. viii+274.

Dr. George-Bruce Halsted. *Géométrie Rationnelle. Traité Elémentaire de la Science de la l'Espace.* Traduction Française par Paul Barbarin. Avec une Préface de C.-A. Laisant. Paris, Gauthier-Villars. 1911. Pp. iv+296.

The French translation by Professor Barbarin of the second edition of Professor Halsted's *Rational Geometry* is a handsome volume published by the book firm of Gauthier-Villars. The book is prefaced by Professor C. A. Laisant, who is well known to American students of mathematics. It is now seven years since the publication of the first edition of this work, a review of which by Professor Davisson of Indiana University was published in the *Bulletin of the American Mathematical Society,* Vol. XI, pp. 330-336.

Poincaré has characterized Halsted's introduction of Hilbert's ideas and principles in an elementary text-book as follows:

> "Introduire ce principe dans l'enseignement, c'est bien pour le coup rompre les ponts avec l'intuition sensible, et c'est là je l'avoue, une hardiesse qui me parait presque une témérité."

Daring and rash it may be, but Professor Halsted certainly deserves credit for his temerity, for he has given to American teachers and students of mathematics a well-written text-book on Geometry in which the fruitful ideas and principles of Hilbert have been most ably interpreted and applied. To what extent the book can be used in elementary instruction, as a high-school text-book, for instance, the writer is not prepared to discuss, but it seems reasonable to suppose that it can be taught with the best results to students who have had a preliminary course in geometry such as the one published some time ago by E. Borel,* in which intuitional methods are largely used and formal demonstrations relegated to a second place. The prevalent American custom of introducing a student to formal demonstrative geometry without any preceding intuitional geometry seems pedagogically unwise and is largely responsible for the distaste of the average high-school student for mathematical subjects. The century-long authority of Euclid will of course prevent the book from becoming popular or a good seller. To the average teacher of mathematics with limited training and knowledge of modern mathematics it will seem revolutionary and strange because it is new, not because it is inherently more difficult than Euclid or any of his modern followers. In some respects it is simpler. With Euclid the theory of proportion involves the difficult idea of limits and continuity (commensurability); here we have a sect-calculus (Ch. VII) based on very simple assumptions and on Schur's very elementary proof of Pascal's theorem. The vexing question about definitions of point, straight line, plane, is solved by not defining them at all, but assuming them as the "elements" or "things" that form the material of the structure.*

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* E. Borel, *Géométrie, Premier et second cycles.* Paris, Armand Colin, 1905,

It is not our purpose to review each particular chapter in detail. The changes which have been made in this second and revised edition are all decided improvements. The sect-calculus is introduced by Schur's proof of Pascal's theorem on the basis of which the commutative, associative and distributative laws for addition (subtraction), multiplication, and division of sects are proved. This seems preferable to the method of the first edition, which made use of the circle.

The author tries to avoid as much as possible the use of the Archimedes assumption. In the first edition he was disposed to eliminate it altogether, being satisfied with a mere statement of it. However, in Ch. XI on "Length and Content of the Circle" it was found impossible to avoid the use of this assumption and — a consequence of it — the assumption of a unique sect equal to the length of the circle. From the standpoint of rigor this seems satisfactory, although it is not made quite clear in the text how the Archimedes assumption is involved in the assumption: "There is one and only one sect greater than the perimeter of any inscribed polygon and less than the perimeter of any circumscribed polygon, namely, the length of the circle." The real content of the Archimedes assumption, in so far as it applies to this postulate, is thus left as an unexplained mystery. Perhaps this is the only way out of the dilemma; to go into details might also necessitate saying something about the "axiom of completeness" and this would have been decidedly beyond the scope of an elementary book. On the whole we think the author has skilfully avoided the difficulties of this part of the subject without sacrificing rigor.

In writing an elementary text-book based on Hilbert's system of axioms a difficulty presents itself that can only be conquered by evading it. Thus, in the first edition of Hilbert's *Foundations* we find an axiom II, 4 as follows: "Any four points on a straight line can always be so lettered, A, B, C, D, that B is between A and C and also between A and D, and furthermore C is between A and D and also between B and D." This assumption was later found to be a demonstrable theorem,† but we note that the proof is rather long and difficult for beginners and the author has therefore rightly placed it in appendix I. The student will readily grant the proposition, appeal being made to intuition, but we have here a striking illustration of the difficulty we meet when any appeal to intuition is denied, as it must be in a Rational Geometry.

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* We note that the "National Committee of Fifteen on Geometric Syllabus" seems to take a similar view: "Certain concepts are so elementary that no simple terms exist by which to define them, although they can easily be explained. For example, *point, line, surface, space, angle, straight line, curve.* The committee recommends that teachers give more attention to instilling a clear concept of such terms and none to exact definitions." This is all very good indeed, but we find on page 30 of the same report among the general list of *postulates:* "A straight line is the shortest line between two points." First the student is told that a straight line is of illimitable extent, and then he is told to believe that such a line is the shortest line. This is truly what Max Dehn calls "Ein energischer widerspruch."

† Proved by R. L. Moore and E. H. Moore.

And now quite recently in a paper published in the *Annalen*, B. 71, 1911, Rosenthal[*] has proved that the system of assumptions of congruence can be simplified; he shows that assumption III, 5[†] is wholly superfluous, the same being true of parts of III, 1 and III, 4. His proofs of these superfluous axioms are not very simple in an elementary sense, and in a future third edition of Halsted's geometry we shall perhaps have an appendix II devoted to these theorems, the proofs of which we hope the author will simplify sufficiently for elementary use. It follows also from Rosenthal's work that the five congruence assumptions for the geometry on the sphere (*Pure Spherics*, Ch. XVI, pp. 226-227, French edition) may be reduced by one and assumptions III, 1 and III, 4 simplified so as to correspond to the same group of assumptions for the plane.

In appendix II the author introduces the basic assumptions for the compasses. Here we find a notable improvement in the present edition, the two assumptions being reduced to one, viz.: "If a straight have a point within a circle, it has two points on the circle." Schur has pointed out that this assumption can be proved by means of the Archimedes assumption. The author wisely omits such proof, but he might have stated the fact, since the student easily gets an idea that this is an entirely new assumption independent of the rest.

A word may here be said about the nomenclature. Professor Halsted's innovations have not been generally accepted by text-book writers, but that proves nothing for or against his choice of terms. The word 'co-straight' will be objected to by purists as a hybrid; 'co-punctal' is better than 'concurrent,' if the idea of motion is to be eliminated; while the word 'sect' seems preferable to 'segment,' which is used to designate something else. The French translator has rendered 'sect' by 'segment,' 'co-straight' has for its equivalent 'collineaire,' and 'co-punctal' is translated 'coponctuelle.' We note with pleasure that the "Committee of Fifteen" has been kind enough to allow the use of the word "congruent." It may be that the author of *Rational Geometry* will live to see the day when a future more liberal "Committee of Fifteen" shall accept his nomenclature, if not *in toto*, yet the greater number of his terms.

As regards print, binding, and general appearance, the French edition is a real work of art. We shall say nothing about Professor Barbarin's translation; a competent judge, Professor Laisant, says: "Professor

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[*] *Vereinfachungen des Hilbertschen System der Kongruenzaxiome*, von Arthur Rosenthal in Munchen.

[†] Assumption III, 5 is: "If an angle (h, k) is congruent as well to the angle (h', k') as also to the angle (h'', k'') then is also the angle (h', k') congruent to the angle (h'', k'').

Halsted's book is lucidly written, the translation is at once faithful and smooth."

We congratulate Professor Halsted on the well-merited success that his book has attained. Sooner or later it will no doubt exercise a profound influence on the teaching of geometry in our country.

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A METHOD FOR THE SOLUTION OF SIMULTANEOUS QUADRATIC EQUATIONS OF THE SYMMETRIC TYPE.

By B. E. MITCHELL, Nashville, Tennessee.

The following method is suggested but not developed in Fine's *College Algebra.*

The most general quadratic of the symmetric type between two variables is $a(x^2+y^2)+2bxy+c(x+y)+d=0$.

Let x and y be the roots of $u^2+pu+q=0$. Then

$$u=\begin{cases} x=\frac{1}{2}[-p\pm\sqrt{(p^2-4q)}] \\ y=\frac{1}{2}[-p\mp\sqrt{(p^2-4q)}] \end{cases}\ldots(\mathrm{I}).$$

The double sign is used in each because they are the roots of a symmetric equation.

From the theory of quadratic equations we have

$$x+y=-p\ldots(1),$$

and $$xy=q\ldots(2),$$

from which we calculate $$x^2+y^2=p^2-2q\ldots(3).$$

These are the three symmetric forms that occur in the general equation above, and are the only ones of the second degree.

Let us solve two simultaneous equations by this method.

$$\begin{cases} a_1(x^2+y^2)+2b_1xy+c_1(x+y)+d_1=0\ldots(1). \\ a_2(x^2+y^2)+2b_2xy+c_2(x+y)+d_2=0\ldots(2). \end{cases}$$

Then

$$\begin{cases} a_1p^2-2(a_1-b_1)q-c_1p+d_1=0\ldots(3). \\ a_2p^2-2(a_2-b_2)q-c_2p+d_2=0\ldots(4). \end{cases}$$

Each of the equations (3) and (4) is linear in q. Solving them for q, we have